Readers' Forum

Comment on "Improved Series Solutions of Falkner-Skan Equation"

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IN Ref. 1, Afzal presents solutions of the Falkner-Skan equation

$$f'' + ff'' + \beta(' - f'^2) = 0 \quad \text{(1a)}$$

$$f(0) = f'(0) = 0, \qquad f'(\infty) = 1 \quad \text{(1b)}$$

in terms of expansions in powers of β. It is the purpose of this Comment to show that this is a particulary difficult and awkward approach to a mathematically trivial problem. Elements of the exploitation of the contraction mapping properties of the Falkner-Skan equation viewed as an operator on f'' were first noted by Weyl[2-4] and fully exploited in the solution of self-similar, binary gas, compressible, laminar boundary layers by Wortman.[5] In the present case, when an integrating factor

$$E = \exp\left(\int_0^\eta f \mathrm{d}\eta\right) \quad (2)$$

is used, formal integration of Eq. (1a) yields

$$f'' = E^{-1}\left[K - \beta\int_0^\eta (1 - f'^2)E\mathrm{d}\eta\right] \quad (3)$$

with K being the constant of integration determined from the integration of Eq. (3) between the limits of 0 and ∞. Thus,

$$K = \left[1 - \beta\int_0^\infty E^{-1}\int_0^\eta (1 - f'^2)\mathrm{d}x\mathrm{d}\eta\right] \Big/ \int_0^\infty E^{-1}\mathrm{d}\eta \quad (4)$$

and f' and f are obtained from successive quadratures. The iteration sequence may be started from an arbitrary f'' and Simpson's rule or trapezoidal integration is sufficient for five-place accuracy in $f''(0)$. Equation (3) is, in fact, a contraction map for moderate values of β and the iteration sequence is stabilized through weighted averaging up to $\beta = 20$.[6] Three-dimensional foreign gas injection applications and three-dimensional turbulent flows were exhibited in Refs. 7 and 8, respectively. In all cases, extremely simple compact computer codes were used and convergence was achieved in about 1 s of IBM 370 computer time. The basic Falkner-Skan equation can be programmed in about 24 lines of code.

The efforts of Ref. 1 represent exemplary industriousness worthy of admiration, but the contribution to a problem-solving-oriented profession such as engineering is not obvious, and thus the place of the Note in the *AIAA Journal* must be questioned.

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Reply by Author to A. Wortman

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IN his Comment, Wortman has attempted a comparison of the time and effort involved in improving the convergence of the series solution[1] of the Falkner-Skan equation due to Aziz and Na[2] with purely numerical solutions. The technique of computer-extended series[1,2] is relatively recent. In a recent review on the subject, Van Dyke[3] has concluded that "clearly the technique of computer-extended series is not yet ready to be applied routinely. Until we understand it better it should be regarded as a useful but dangerous tool, to be used with utmost care and skepticism."

For the Falkner-Skan equation, the eleven-term series solution in powers of β was improved[2] for $-0.19884 \le \beta \le 2$ by the application of the Shanks transformation, which does not exploit the analytical structure of the series. If the series permits an estimate of the analytical structure, there are more effective ways of improving the series.[4] Therefore, it was logical to explore the analytical structure of Aziz and Na's[2] series (location and nature of the nearest singularity), which formed a natural basis for the improvement of its convergence, once and for all values of β. Recasting of the original series[2] by Euler transformation or by subtraction of the singularity carried out by Afzal[1] is not only fairly simple and straightforward but also predicts very good results for all values of β in the range $-0.19884 \le \beta \le \infty$.

Regarding the so-called mathematically trivial Falkner-Skan equation, the work of Cebeci and Keller[5] (where the

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